

AP Calculus AB

<i>COURSE OUTLINE</i>		
Unit One	<i>Limits</i> <i>Limit of function graphically and numerically</i> <i>Limit Laws</i> <i>Limits and Asymptotes</i> <i>Definition of Derivative</i>	20 periods
Unit Two	<i>Derivatives</i> <i>Polynomial, Product and Quotient Rule</i> <i>Trigonometric Functions Chain Rule</i> <i>Implicit Differentiation, Logarithmic Functions</i> <i>Rates of Change</i> <i>Related Rates</i> <i>Linear Approximation</i>	23 periods
Unit Three	<i>Applications of Derivatives</i> <i>Minimum and Maximum Values</i> <i>The Mean Value Theorem</i> <i>Derivatives Affect Shape of Graph</i> <i>L'Hospital's Rule</i> <i>Curve Sketching</i> <i>Optimization</i> <i>Newton's Method</i> <i>Antiderivatives</i>	25 periods
Unit Four	<i>Integrals</i> <i>Areas and Distances</i> <i>Definite Integral</i> <i>Fundamental Theorem of Calculus</i> <i>Indefinite Integrals</i> <i>Substitution Rule</i> <i>Trigonometric Integrals</i> <i>Trigonometric Substitution</i> <i>Approximate Integration (trapezoidal rule)</i>	25 periods
Unit Five	<i>Applications of Integrals</i> <i>Area Between Curves</i> <i>Volumes</i> <i>Work</i> <i>Average Value of a Function</i>	18 periods
Unit Six	<i>Differential Equations</i> <i>Modeling with Differential Equations</i> <i>Direction Fields and Euler's Method</i> <i>Separable Equations</i> <i>Models for Population Growth</i> <i>Linear Equations</i>	10 periods

School-wide Academic Expectations Taught In This Course

- Analysis
- Collaboration
- Communication
- Literacy

School-wide Social and Civic Expectations Taught in This Course

- Demonstrate Resiliency
- Demonstrate Responsibility
- Demonstrate Respect

Mathematical Practices and Skills for AP Calculus (MPACs)

1. Implementing Mathematical Processes: Determine expressions and values using mathematical procedures and rules
 - 1A Identify the question to be answered or problem to be solved (not assessed).
 - 1B Identify key and relevant information to answer a question or solve a problem (not assessed).
 - 1C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).
 - 1D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.
 - 1E Apply appropriate mathematical rules or procedures, with and without technology.
 - 1F Explain how an approximated value relates to the actual value.
2. Connecting Representations: Translate mathematical information from a single representation or across multiple representations.
 - 2A Identify common underlying structures in problems involving different contextual situations.
 - 2B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.
 - 2C Identify a re-expression of mathematical information presented in a given representation.
 - 2D Identify how mathematical characteristics or properties of functions are related in different representations.
 - 2E Describe the relationships among different representations of functions and their derivatives.
3. Justification: Justify reasoning and solutions.

- 3A Apply technology to develop claims and conjectures (not assessed).
- 3B Identify an appropriate mathematical definition, theorem, or test to apply.
- 3C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.
- 3D Apply an appropriate mathematical definition, theorem, or test.
- 3E Provide reasons or rationales for solutions and conclusions.
- 3F Explain the meaning of mathematical solutions in context.
- 3G Confirm that solutions are accurate and appropriate.

4. Communication and Notation: Use correct notation, language, and mathematical conventions to communicate results or solutions.

- 4A Use precise mathematical language.
- 4B Use appropriate units of measure.
- 4C Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y' , dy and dx). 4.D Use appropriate graphing techniques.
- 4E Apply appropriate rounding procedures

Unit 1: Limits

Introduction and Established Goals: Many calculus concepts are developed by considering a discrete model and then the consequences of a limiting case. Therefore, the idea of limits is essential for discovering and developing important ideas, definitions, formulas, and theorems in calculus. Students must have a solid intuitive understanding of limits and be able to compute various limits, including one-sided limits, limits at infinity, the limit of a sequence, and infinite limits. They should be able to work with tables and graphs in order to estimate the limit of a function at a point. Students should know the algebraic properties of limits and techniques for finding limits of indeterminate forms, and they should be able to apply limits to understand the behavior of a function near a point. Students must also understand how limits are used to determine continuity, a fundamental property of function.

Desired Outcome(s): Students will be able to find the limit of a function graphically and analytically and apply limits to determining continuity of a function.

Essential Question(s): How can the concept of a limit be used to understand the behavior of functions? How do you find the limit of a function given a graph, table, or equation? How are limits used to determine continuity of a function?

Key Terms/Concepts:

Secant line, tangent line, average velocity, instantaneous velocity, slope, equation of tangent line, one-sided limits, left-hand limit, right-hand limit, infinite limits, vertical asymptote, limit laws, direct substitution property, the squeeze theorem, continuous, discontinuous, removable discontinuity, infinite discontinuity, jump discontinuity, The Intermediate Value Theorem, horizontal asymptote, derivative, differentiation, vertical tangent line, corner, second derivative, third derivative, velocity, acceleration, jerk

LEARNING PLAN

TANDARD	LEARNING OBJECTIVES (Content and Skill)	STRUCTIONAL STRATEGIES	ASSESSMENT EVIDENCE
	<ol style="list-style-type: none"> 1. Express limits symbolically using correct notation. 2. Interpret limits expressed symbolically. 3. Estimate limits of functions. 4. Determine limits of functions. 5. Deduce and interpret behavior of functions using limits. 6. Identify the derivative of a function as the limit of a difference quotient. 	<ul style="list-style-type: none"> o The Tangent and Velocity Problems Guided Notes o The Limit of a Function Guided Notes o Limit Laws Guided Notes o Limits at Infinity Guided Notes o Derivatives and Rates of Change Guided Notes o Derivative as a Function Guided Notes o Practice Problem Set per topic o Daily Warm-up Activity o Kahoot review on limits 	<p>Quizzes per section</p> <p>Khan Academy Practice Sets, Quizzes, Tests</p> <p>Kahoot, Quizizz, Quizlet</p> <p>Unit Test</p>

		<ul style="list-style-type: none"> o Quizizz review on limits o Guide to Limits 	
	<ul style="list-style-type: none"> 7. Analyze functions for intervals of continuity or points of discontinuity. 8. Determine the application of important calculus theorems using continuity. 9. Recognize the connection between differentiability and continuity. 	<ul style="list-style-type: none"> o Continuity Guided Notes o Kahoot review on continuity o Quizizz review on continuity o Guide to Continuity 	

Suggested Resources and Texts: Single Variable Calculus 7E, AP Central website, Paul’s Online Notes, Calculus.org website, Khan Academy, Mrs. Reyes’ Calculus videos

Suggested Technology: TI-84 plus calculator and TI-89 calculator

Unit 2: Derivatives

Introduction and Established Goals: Using derivatives to describe the rate of change of one variable with respect another variable allows students to understand change in a variety of contexts. Students will build the derivative using the concept of limits and use the derivative primarily to compute the instantaneous rate of change of a function. Students should be able to use the different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. Students will find the derivative of a function using all rules and methods and look at some applications of derivatives.

Desired Outcome(s): Students will be able to find the derivative of a function algebraically and use it to solve application problems.

Essential Question(s): How do you find the derivative of a function? How can derivatives be used to solve problems?

Key Terms/Concepts: constant function, power function, power rule, constant multiple rule, sum rule, difference rule, exponential function, product rule, quotient rule, trigonometric functions, chain rule, implicit differentiation, inverse trigonometric functions, logarithmic functions, rates of change, exponential growth and decay, related rates, linear approximation, tangent line approximation, linearization, differentials

LEARNING PLAN

STANDARD	LEARNING OBJECTIVES (Content and Skill)	INSTRUCTIONAL STRATEGIES	
	<ol style="list-style-type: none"> 1. Identify the derivative of a function as the limit of a difference quotient. 2. Estimate derivatives. 3. Calculate derivatives. 4. Determine higher order derivatives. 	<ul style="list-style-type: none"> o Derivatives of Polynomials and Exponential Functions Guided Notes o The Product and Quotient Rule Guided Notes o Derivatives of Trigonometric Functions Guided Notes o Derivatives of Inverse Functions Guided Notes and Direct Instruction o The Chain Rule Guided Notes o Implicit Differentiation Guided Notes o Derivatives of Logarithmic Functions Guided Notes o Kahoot review on derivatives o Quizizz review on derivatives o Function and Their Derivative Matching Activity 	<p>Quizzes per section</p> <p>Khan Academy Practice Sets, Quizzes, Tests</p> <p>Kahoot, Quizizz, Quizlet</p> <p>Unit Test</p>

	<ol style="list-style-type: none"> 5. Interpret the meaning of a derivative within a problem. 6. Solve problems involving the slope of a tangent line. 7. LO 2.3C: Solve problems involving related rates. 8. LO 2.3D: Solve problems involving rates of change in applied contexts. 	<ul style="list-style-type: none"> o Rates of Change in Natural and Social Science Guided Notes o Exponential Growth and Decay Guided Notes o Related Rates Video Guided Notes o Linear Approximations and Differentials Guided Notes o Practice Problem Set per topic o Daily Warm-up Activity 	

Suggested Resources and Texts: Single Variable Calculus 7E, AP Central website, Paul’s Online Notes, Calculus.org website, Khan Academy, Mrs. Reyes’ Calculus videos

Suggested Technology: TI-84 plus calculator and TI-89 calculator

Unit 3: Applications of Derivatives

Introduction and Established Goals: Using derivatives to describe the rate of change of one variable with respect another variable allows students to understand change in a variety of contexts. Students will build on their knowledge of finding the derivative of a function and use it to solve problems. Applications of the derivative include finding the slope of a tangent line to a graph at a point, analyzing a function (finding intervals of increase, decrease, finding maximum and minimum values, intervals of concavity), and solving problems involving rectilinear motion. Students should be able to use the different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. In addition, students should be able to apply the Mean Value Theorem and be familiar with a variety of real-world applications including related rates, optimization, and growth and decay.

Desired Outcome(s): Students will be able to use derivatives in a variety of applications.

Essential Question(s): How can derivatives be used to solve application problems?

Key Terms/Concepts: Absolute maximum, absolute minimum, local maximum, local minimum, Extreme Value Theorem, Fermat’s Theorem, critical number, Closed Interval Method, Rolle’s Theorem, The Mean Value Theorem, Increasing/Decreasing test, First Derivative Test, concave up, concave down, concavity test, inflection point, Second Derivative Test, indeterminate form, l’Hospital’s Rule, curve sketching, optimization, Newton’s method, antiderivative

LEARNING PLAN

TANDARD	LEARNING OBJECTIVES (Content and Skill)	STRUCTIONAL STRATEGIES	ASSESSMENT EVIDENCE
	<ol style="list-style-type: none"> 1. Use derivatives to analyze properties of a function. 2. Interpret the meaning of a derivative within a problem. 3. Solve problems involving the slope of a tangent line. 4. Solve problems involving optimization, and rectilinear motion. 	<ul style="list-style-type: none"> o Maximum and Minimum Values Guided Notes o The Mean Value Theorem Guided Notes o How Derivatives Affect the Shape of a Graph Guided Notes o Indeterminate Forms and l’Hospital’s Rule Guided Notes o Summary of Curve Sketching Guided Notes o Optimization Guided Notes o Newton’s Method Guided Notes o Antiderivatives Guided Notes 	<p>Quizzes per section</p> <p>Khan Academy Practice Sets, Quizzes, Tests</p> <p>Kahoot, Quizizz, Quizlet</p> <p>Unit Test</p>
	<ol style="list-style-type: none"> 5. Solve problems involving rates of 	<ul style="list-style-type: none"> o Matching graphs activity 	

	change in applied contexts. 6. Apply the Mean Value Theorem to describe the behavior of a function over an interval.	<ul style="list-style-type: none"> o Practice Problem Set per topic o Daily Warm-up Activity 	

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Suggested Technology: TI-84 plus calculator and TI-89 calculator

Unit 4: Integrals

Introduction and Established Goals: Integrals are used in a wide variety of practical and theoretical applications. Students should understand the definition of a definite integral involving a Riemann Sum, be able to approximate a definite integral using different methods, and be able to compute definite integrals using geometry. They should be familiar with basic techniques of integration and properties of integrals. It is crucial that students grasp the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus. Students should be able to work with and analyze functions defined by an integral.

Desired Outcome(s): Students will be able to find indefinite and definite integrals using a variety of methods.

Essential Question(s): How do you find definite and indefinite integrals? How do you use integrals to solve application problems?

Key Terms/Concepts: Area problem, sample points, definite integral, Riemann Sum, midpoint rule, right endpoints, left endpoints, Fundamental Theorem of Calculus, Net Change Theorem, substitution rule

LEARNING PLAN

TANDARD	LEARNING OBJECTIVES (Content and Skill)	STRUCTIONAL STRATEGIES	ASSESSMENT EVIDENCE
	<ol style="list-style-type: none"> 1. Recognize antiderivatives of basic functions. 2. Interpret the definite integral as the limit of a Riemann sum. 3. Express the limit of a Riemann sum in integral notation 4. Approximate a definite integral. 	<ul style="list-style-type: none"> o Areas and Distances Guided Notes and Practice Problems o The Definite Integral Guided Notes o The Fundamental Theorem of Calculus Guided Notes o Indefinite Integrals and the Net Change Theorem Guided Notes o The Substitution Rule Guided Notes 	<p>Quizzes per section</p> <p>Khan Academy Practice Sets, Quizzes, Tests</p> <p>Kahoot, Quizizz, Quizlet</p> <p>Unit Test</p>

		<ul style="list-style-type: none"> o Trigonometric Integrals Guided Notes o Approximate Integration Guided Notes 	
	<ol style="list-style-type: none"> 5. Calculate a definite integral using areas and properties of definite integrals. 6. Analyze functions defined by an integral. 7. Calculate antiderivatives. 8. Evaluate definite integrals. 9. Interpret the meaning of a definite integral within a problem. 	<ul style="list-style-type: none"> o Practice Problem Set per topic o Daily Warm-up Activity 	

Suggested Resources and Texts: Single Variable Calculus 7E, AP Central website, Paul's Online Notes, Calculus.org website, Khan Academy, Mrs. Reyes' Calculus videos

Suggested Technology: TI-84 plus calculator and TI-89 calculator

Unit 5: Applications of Integration

Introduction and Established Goals:

Integrals are used in a wide variety of practical and theoretical applications. Students should understand the definition of a definite integral involving a Riemann Sum, be able to approximate a definite integral using different methods, and be able to compute definite integrals using geometry. They should be familiar with basic techniques of integration and properties of integrals. The interpretation of a definite integral is an important skill, and students should be familiar with area, volume, and motion applications, as well as with the use of the definite integral as an accumulation function.

Desired Outcome(s): Students will be able to find the area between curves and the volume of a solid created by revolution. Students will be able to use the Fundamental Theorem of Calculus to solve application problems, including finding the average value of a function and problems involving force and work.

Essential Question(s): How do you find the area between two curves? How do you find the volume of any solid? How do you find the average value of a function?

Key Terms/Concepts:

Volume, solids of revolution, work, force, average value of function, base, disc, washer, cross-section

LEARNING PLAN

STANDARD	LEARNING OBJECTIVES (Content and Skill)	INSTRUCTIONAL STRATEGIES	ASSESSMENT EVIDENCE
	<ol style="list-style-type: none"> 1. Recognize antiderivatives of basic functions 2. Calculate antiderivatives. 3. Evaluate definite integrals. 	<ul style="list-style-type: none"> o Area Between Curves Guided Notes o Volumes Guided Notes o Average Value of a Function Guided Notes o Practice Problem Set per topic o Daily Warm-up Activity o Volume Cheat Sheet 	<p>Quizzes per section</p> <p>Khan Academy Practice Sets, Quizzes, Tests</p> <p>Kahoot, Quizizz, Quizlet</p> <p>Unit Test</p>
	<ol style="list-style-type: none"> 4. Apply definite integrals to problems involving the average value of a function. 		

	<ol style="list-style-type: none"> 5. Apply definite integrals to problems involving motion. 6. Apply definite integrals to problems involving area, and volume and length of a curve. 7. Use the definite integral to solve problems in various contexts. 		
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Suggested Resources and Texts: Single Variable Calculus 7E, AP Central website, Paul’s Online Notes, Calculus.org website, Khan Academy, Mrs. Reyes’ Calculus videos, 3D printed models, 3D solids

Suggested Technology: TI-84 plus calculator and TI-89 calculator

Unit 6: Differential Equations

Introduction and Established Goals: Differential equations is one of the most important application of calculus. Students will analyze a differential equation, find an explicit formula when possible, or draw a slope field as a solution to the differential equation. Students will investigate the logistic population model in depth.

Desired Outcome(s): Students will be able to analyze a differential equation, verify a solution, solve by separating, and draw a slope field for a given differential equation.

Essential Question(s): What is a differential equation and what is it used for? How can you use a differential equation to model a situation? What is a directional field?

Key Terms/Concepts: Differential equation, direction fields, separable equations, initial condition, logistic growth model, carrying capacity

LEARNING PLAN

TANDARD	LEARNING OBJECTIVES (Content and Skill)	INSTRUCTIONAL STRATEGIES	ASSESSMENT EVIDENCE
	<ol style="list-style-type: none"> 1. Analyze differential equations to obtain general and specific solutions. 2. Interpret, create, and solve differential equations from problems in context. 	<ul style="list-style-type: none"> o Modeling with Differential Equations Guided Notes o Direction Fields and Euler’s Method Guided Notes o Separable Equations Guided Notes o Models for Population Growth Guided Notes 	<p>Quizzes per section</p> <p>Khan Academy Practice Sets, Quizzes, Tests</p> <p>Kahoot, Quizizz, Quizlet</p> <p>Unit Test</p>

		<ul style="list-style-type: none">o Practice Problem Set per topico Daily Warm-up Activityo Slope Field Matching Activity	
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Suggested Resources and Texts: Single Variable Calculus 7E, AP Central website, Paul's Online Notes, Calculus.org website, Khan Academy, Mrs. Reyes' Calculus videos

Suggested Technology: TI-84 plus calculator and TI-89 calculator