Algebra II

COURSE OUTLINE		
Unit One	*Functions and Inverse Functions*	*30 Days*
Unit Two	*Quadratic Functions*	*25 Days*
Unit Three	*Polynomial Functions*	*30 Days*
Unit Four	*Rational and Power Functions*	*25 Days*
Unit Five	*Exponential and Logarithmic Functions*	*30 Days*
Unit Six	*Sequences and Series*	*20 Days*

School-wide Academic Expectations Taught In This Course

- Analysis
- Collaboration
- Communication
- Literacy

School-wide Social and Civic Expectations Taught in This Course

- Demonstrate Resiliency
- Demonstrate Responsibility
- Demonstrate Respect

Content Standards Taught in This Course

A.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.REI.D.11 Explain why the x-coordinate of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include where f(x) and/or g(x) are linear, rational, absolute value, exponential and logarithmic functions.

A.REI.D.12 Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality) and graph the solution set of a system of linear inequalities as the intersection of the corresponding half-planes.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then *f*(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation y = *f*(x).

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F.IF.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
F.BF.1b	Combine standard function types using arithmetic operations.
F.BF.1c (+)	Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
F.BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
F.BF.4	Find inverse functions.
F.BF.4a	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2(x^3)$ for $x > 0$ or $f(x) = (x + 1)/(x – 1)$ for $x \neq 1$ (x not equal to 1).
F.BF.4b (+)	Verify by composition that one function is the inverse of another.
F.BF.4c (+)	Read values of an inverse function from a graph or a table, given that the graph has an inverse.
A.SSE.3	Choose and produce an equivalent from of an expression to reveal and explain properties of the quantity represented by the expression.
A.SSE.3a	Factor a quadratic expression to reveal zeros of the function it defines.
A.SSE.3b	Complete the square in a quadratic expression to reveal the maximum/minimum value of the function it defines.
A.REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A.REI.4	Solve quadratic equations in one variable.
A.REI.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
BF.A.1	Write a function that describes a relationship between two quantities.
CED.A.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.
CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
F.IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F.IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
F.IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step

functions and absolute value functions**.**

N.CN.1 Know there is a complex number *i* such that $i^2 = \sqrt{-1}$, and every complex number has the form *a* + *bi* with *a* and *b* real.

N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (Note that only functions with real coefficients are considered in this investigation.)

N.RN.3 Explain why the sum or product of two rational numbers is rational and the sum of a rational and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; multiplicity of roots; symmetries; end behavior; and periodicity.

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.APR.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2) \bullet (x^2 + y^2)$.

A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

A.APR.5 (+) Know and apply that the Binomial Theorem gives the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

A.SSE.1b Interpret complicated expressions by viewing one or more parts as a single entity.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship.

F.IF.7d(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable in a modeling context.

A.APR.1b Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for more complicated examples, a computer algebra system.

A.APR.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for more complicated examples, a computer algebra system.

A.APR.7(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous roots may arise.

A.REI.11b Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equations f(x) = g(x); find the solutions approximately, e.g. using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

A-SSE-1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as a product of P and a factor not depending on P.

A-SSE-4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

A-CED-1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED-2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-IF. 7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF-8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-BF-3 Identify the effect on the graph of replacing $f(x)$ by $fk, kf(x), f(kx),$ and $f(x+k)$ for specific values of k (both positive and negative); find the values of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF-5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

F-LE-4 For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where a, c and *d* are numbers and the base *b* is 2, 10 or *e;* evaluate the logarithm using technology.

F-LE-5 Interpret the parameters of an exponential function in terms of a context.

A.REI.11b Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equations f(x) = g(x); find the solutions approximately, e.g. using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*

F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

A-SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

F-BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

SL.9-10.1.c Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions.

Unit 1: Functions and Inverse Functions

Introduction and Established Goals: This Unit serves as both a review and an extension of functions from Algebra 1. Students will be familiar with functions, particularly linear functions, from Algebra 1. These subjects are reviewed in new contexts, such as linear programming and conic sections. The Unit then extends students' understanding of functions by examining the important concepts of function growth, transformation of functions, creating new functions from old, and inverse functions. This Unit sets the groundwork for the rest of Algebra 2 by investigating functions in a general way which is then applied as needed later in the curriculum. As many of the ideas in this Unit review concepts introduced in Algebra 1, there is an emphasis throughout Unit 1 on some of the applications of these ideas to realistic contexts.

Desired Outcome(s):

- A function is a special kind of relation where each member of the domain of the function is associated with exactly one member of the range of the function.
- We can study a very wide range of phenomena by starting with a very small group of "parent" functions and applying transformations to those functions.

CT State Standard(s):

A.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.REI.D.11 Explain why the x-coordinate of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include where f(x) and/or g(x) are linear, rational, absolute value, exponential and logarithmic functions.

A.REI.D.12 Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality) and graph the solution set of a system of linear inequalities as the intersection of the corresponding half-planes.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*.

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F.BF.1b Combine standard function types using arithmetic operations.

F.BF.1c (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.4 Find inverse functions.

F.BF.4a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2(x^3)$ for $x > 0$ or $f(x) = (x + 1)/(x – 1)$ for $x \neq 1$ (x not equal to 1).

F.BF.4b (+) Verify by composition that one function is the inverse of another.

F.BF.4c (+) Read values of an inverse function from a graph or a table, given that the graph has an inverse.

SL.9-10.1.c Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions.

Common Core Standard(s):

- MP5: Use appropriate tools strategically
- MP4: Look for and make use of structure

Essential Question(s):

- What is a function?
- What are the different ways in which functions may be represented?
- How do we find an optimal solution to a linear programming problem?
- What is the meaning of the domain and range of a function?
- What is a family of functions?
- How do different families of functions grow?
- What is the effect of a transformation of an independent variable? What is the effect of a transformation of a dependent variable?
- (+) What does it mean to compose two functions?
- What is the inverse of a function?
- How can functions be used to model real world situations, make predictions, and solve problems?

Key Terms/Concepts: Absolute value function, Algorithm, Angle of a generator, Area of a circle, Axis of symmetry, Boundary line, Circle, Circumference, Composition of functions, Constraint, Dependent variable, Depreciation, Directrix, Domain, Ellipse, Equivalent equation and inequality, Even function, Exponential function, Family of functions, Feasible region, Floor function, Foci of an ellipse, hyperbola, or parabola, Function, Function notation, Greatest integer function, Half-plane, Horizontal and vertical axes, Horizontal line test, Horizontal shift/translation, Hyperbola, Independent variable, Inequality, Input and output of a function, Inside change, Inverse function, Isometry, Line symmetry, Linear growth, Major axis/minor axis of an ellipse, Maximum/minimum, Natural domain, Nonlinear growth, Objective function, Odd function, One-to-one function, Optimization, Ordered pair, Origin, Outside change, Parabola, Parameter, parent function, Piecewise function, Quadratic function, Quadritium, Radius of a circle, Range, Reflection, Relation, Restricted domain, Root function, Rotational symmetry, Shading, Solution of a system of linear equations, Solution of a system of linear inequalities, Standard form, Slope, Step function, Symmetric with respect to the origin, Symmetric with respect to the y-axis, System of equations, System of inequalities, Test point, Transformation, Vertex, Vertical stretch, Vertical shift/translation

LEARNING PLAN

STANDARD	LEARNING OBJECTIVES (Content and Skill)	INSTRUCTIONAL STRATEGIES	ASSESMENT EVIDENCE
A.REI.D.10 A.REI.D.11 A.REI.D.12 SL.9-10.1.c	1. Students will graph the solution set of a linear system of inequalities in two variables as the intersection of the corresponding system of half-planes, and interpret the result. 2. Students will be able to define an objective function for a given context. 3. Students will be able to use the seven step linear programming algorithm to identify optimal solutions to practical problems. 4. Students will be able to use linear programming to identify optimal solutions to practical problems.	○ Activity 1.1.1 Keeping the Peace ○ Activity 1.1.2 Peacekeeping Problem: Area Constraint ○ Activity 1.1.3: Finding the Objective Function ○ Activity 1.1.4 Exploring Corner Points ○ Activity 1.1.5 The Rationale for Only Evaluating at Corner Points ○ Activity 1.1.6 Meera's Jobs ○ Activity 1.1.7 The Farmer ○ Activity 1.1.8 Farm Subsidies ○ Activity 1.1.9 Natasha's Cat ○ Activity 1.1.10 Additional Linear Programming Problems	Exit Slip 1.1 Quiz
F.IF.1 F.IF.2 F.IF.5 SL.9-10.1.c	5. Be able to determine if a given relation is a function or not. 6. Given a function, be able to determine its domain and range. 7. Be able to represent a function by an equation, table, graph, or verbal description and move	○ Activity 1.2.1 Function Review ○ Activity 1.2.2 What's Reasonable? ○ Activity 1.2.3a Function Junction ○ Activity 1.2.3b Is the Function 1-1? ○ Activity 1.2.4 Are Conics Functions?	Exit Slip 1.2

	comfortably from one representation to another. 8. Determine the reasonableness of the domain and range of a function in a realistic context. 9. Determine whether a function is a 1-1 function from a graph or table of values. 10. Determine whether the basic conic sections are functions or not.		
F.IF.7B SL.9-10.1.c	11. Be able to identify different types of growth using a table 12. Be able to graph and analyze the absolute value function 13. Be able to graph and analyze the piecewise and step functions 14. Understand end behavior of even and odd functions	○ Activity 1.3.1 Linear and Nonlinear Growth ○ Activity 1.3.2 The Absolute Value Function ○ Activity 1.3.3 Piecewise and Step Functions ○ Activity 1.3.4 Symmetry Review ○ Activity 1.3.5 Even and Odd Functions	Exit Slip 1.3.1 Exit Slip 1.3.2 Quiz
F.BF.1b F.BF.3 SL.9-10.1.c	15. Given two functions f(x) and g(x), be able to find f + g, f – g, fg, and f ÷ g symbolically and for f + g and f – g graphically. 16. Represent a verbal description of a function transformation symbolically. 17. Understand the difference between a transformation of an independent variable and a dependent variable. 18. Given a function f(x), be able to describe the effects of the transformations f(x) + k,	○ Activity 1.4.1 Putting Functions Together (Pt 1 and 2) ○ Activity 1.4.2 Inside Change, Outside Change (Pt 1 and 2) ○ Activity 1.4.3 Move It! Pt 1 ○ Activity 1.4.4 Stretch It! Pt 1 ○ Activity 1.4.5 Transformations Made Easy ○ Activity 1.4.6 Greenhouse Gas Emissions	Exit Slip 1.4

	f(x + k), and kf(x) for a constant k. 19. Given a graph of a function and a transformation of that function, be able to determine the transformation that is represented in the graph. 20. Given a graph of a function, be able to graph (by hand) a transformation of that function.		
F.BF.1c (+) SL.9-10.1.c	21. Students will be able to compose functions using one or more arithmetic operation	○ Activity 1.5.1 Composing Composite Functions ○ Activity 1.5.2 Domains and Graphs of Composite FUNctions ○ Activity 1.5.3 Composition of Function Applications	Exit Slip 1.5
F.BF.4 SL.9-10.1.c	22. Students will verify which functions have inverses 23. Students will use composition techniques to find equations of inverse functions	○ Activity 1.6.1 Functions and their Inverses ○ Activity 1.6.2 Finding Inverses ○ Activity 1.6.3 Using Functions and their Inverses ○ Activity 1.6.4 Are You My Inverse?	Exit Slip 1.6
F.BF.4 SL.9-10.1.c	24. Students will find equations for the inverses of power functions	○ Activity 1.7.1 Review of Exponents ○ Activity 1.7.2 Inverses of Power Functions ○ Activity 1.7.3 Tuning Up	Exit Slip 1.7
			Performance Task
			End of Unit Test

Suggested Resources and Texts:

- https://screen.yahoo.com/u-troops-head-ebola-hot-163128163.html? Activity 1.1.1
- Green and red pencils for plotting the solutions and non solutions in Activity 1.1.1, and Activity 1.1.2A and 1.1.2B
- http://people.richland.edu/james/ictcm/2006/slope.html (1.1.5)
- Belgian Chocolates and Tomato Farmers videos (produced by Mathematics to Enhance Economics: Enhancing Teaching and Learning – at www.metalproject.co.uk/METAL/Resources/Films/linear_programming/index.html (1.6)
- The following website may be used to give insight into who is using linear programming and why: http://www.ilog.com/products/optimization/. (anytime after 1.1.5)

- See www.hsor.org for many high school linear programming problems and information on linear programming (1.5)
- Contemporary College Algebra: Data, Functions, Modeling, 6th ed., McGraw – Hill Primis Custom Publishing . Don Small author (Farm Subsidy problem, 1.1.9)
- Farm subsidy data base at http://farm.ewg.org/farm/region.php?fips=09003 (1.6)
- video on Operational Research use in 1.6 or later- http://www.bnet.com/2422-13950_23-178846.html
- http://illuminations.nctm.org (supplemental problem #3)
- http://www.wikihow.com/Make-a-Duct-Tape-Wallet http://www.ducttapefashion.com/products/prod01.htm (supplemental problem #4)
- TI Numb3rs Linear Programming Activity at http://education.ti.com/educationportal/activityexchange/Activity.do?cid=US&aId=7508 (supplemental problem #6)
- Bakers Choice published by Key Curriculum Press (supplemental problem #7)
- Information relevant to George Dantzig: http://www.lionhrtpub.com/orms/orms-8-05/dantzig.html and http://www.lionhrtpub.com/orms/orms-6-05/dantzig.html (Suggested research question #4)

Suggested Technology: Graphing calculator, Projector, Access to internet

Unit 2: Quadratic Functions

Introduction and Established Goals: Quadratic functions are the first family of functions that students will examine in depth in this Algebra 2 curriculum. Students will apply their knowledge of the effects that different transformations have on the graph of a function to the study of quadratic functions. In particular, students will examine the effects of vertical and horizontal shifts, vertical stretches, and reflections over the x-axis and their effects on properties of quadratic functions.

Mastery of the unit will require students to develop their understanding of: (1) properties of quadratic functions based on the parameters a, b and c in the standard form of the quadratic function $f(x) = ax^2 + bx + c$, and the parameters a, h and k in the vertex form of the quadratic function $f(x) = a(x-h)^2 + k$; (2) different methods for solving quadratic equations; (3) the nature of the roots of a quadratic equation based on the value of the discriminant; (4) the imaginary number i and operations involving complex numbers (5) complex conjugates as solutions of quadratic equations with a negative discriminant; (6) the Fundamental Theorem of Algebra in relation to quadratic functions; (7) modeling using quadratic functions; and (9) solving radical equations.

This unit builds on Unit 8 of the Common Core Algebra 1 Curriculum, providing students a deeper conceptual and procedural understanding of quadratic functions. The coverage of material in this unit will depend on the extent to which students studied and acquired quadratic function competencies in Algebra I. Throughout this unit, authors will highlight the connections between activities in this unit and activities in Unit 8 of the Algebra 1 curriculum, detailing the specific Algebra 1 activities that serve as the foundation of Algebra 2 activities.

Desired Outcome(s):

- Students will understand quadratic functions can be used to model real world relationships and the key points in quadratic functions have meaning in the real world context.
- Students will use dynamic software, graphing calculators, and other technology to explore and deepen their understanding of mathematics.

CT State Standard(s):

A.SSE.3 Choose and produce an equivalent from of an expression to reveal and explain properties of the quantity represented by the expression.

A.SSE.3a Factor a quadratic expression to reveal zeros of the function it defines.

A.SSE.3b Complete the square in a quadratic expression to reveal the maximum/minimum value of the function it defines.

A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A.REI.4 Solve quadratic equations in one variable.

A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

BF.A.1 Write a function that describes a relationship between two quantities.

CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.

CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.IF.C.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions**.**

N.CN.1 Know there is a complex number *i* such that $i^2 = \sqrt{-1}$, and every complex number has the form *a* + *bi* with *a* and *b* real.

N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (Note that only functions with real coefficients are considered in this investigation.)

N.RN.3 Explain why the sum or product of two rational numbers is rational and the sum of a rational and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

SL.9-10.1.c Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions.

Common Core Standard(s):

- MP 3: Construct viable arguments and critique the reasoning of others.
- MP 4: Model with mathematics
- MP 7: Look for and make use of structure

Essential Question(s):

- How do I describe quadratic functions in terms of transformations?
- How do I describe quadratic functions in terms of properties such as concavity and end behavior?
- How do I solve a quadratic equation?
- How can I determine the nature of the roots of a quadratic equation based on the value of the discriminant?
- What is the definition of the imaginary number *i*?
- What is the structure of the complex numbers?
- What is the Fundamental Theorem of Algebra, and what is its relation to quadratic functions?
- How do I use quadratic functions as mathematical models?
- How do I solve radical equations?

Key Terms/Concepts:

Absolute value, Axis of symmetry, Binomial, Binomial expansion, Closed sets, Closure of sets under an operation, Completing the square, Complex conjugate, Complex number, Concave up/down, Cost function, Decreasing function, Distributive property, End behavior, Extraneous root, Extraneous solution, Factor, Factor a quadratic over the integers, Family of functions, Imaginary number, Increasing function, Infinity, Inside change, Mathematical model(ing), Modeling diagram, Monomial, Outside change, Parabola, Parabolic, Parametric equations, Parent function, Piecewise defined function, Profit function, Quadratic formula, Quadratic function, Quadratic regression, Radical equation, Radicand, Revenue function, Root, Sets of numbers {Natural numbers, Whole numbers, Integers, Rationals, Irrationals, Real numbers, Complex numbers}, Slope, Solution, Square root equation, Standard form of a quadratic function, Transformation, Vertex, Vertex form of a quadratic function, x-intercept, y-intercept, zeros of a function.

LEARNING PLAN

STANDARD	LEARNING OBJECTIVES (Content and Skill)	INSTRUCTIONAL STRATEGIES	ASSESEMENT EVIDENCE
F.IF.4 F.IF.7 F.IF.7a	1. Students will be able to describe the graphs of $x^2 + k$, $(x + k)^2$, kx^2, $ax^2 + bx + c$, and $a(x - h)^2 + k$, based on the values of k, a, b, c, and h. 2. Given a quadratic function, students will be able to describe the features of the function (intercepts, intervals where it is positive/negative, intervals where it is increasing and decreasing, concavity, axis of symmetry, and end behavior).	○ Activity 2.1.1: Move It! Part Two ○ Activity 2.1.2: How to Move It! With Standard and Vertex Forms ○ Activity 2.1.3: Stretch It! Part Two ○ Activity 2.1.4: How to Stretch It! With Standard and Vertex Forms ○ Activity 2.1.5: How do quadratic functions behave?	Exit Slip 2.1.1 Exit Slip 2.1.2 Journal Prompt 1
A.SSE.3 A.SSE.3A A.SSE.3B A.REI.4 A.REI.4B F.IF.7B	3. Students will solve quadratic equations using the following methods: graphing, square root method, factoring, completing the square, and the quadratic formula 4. Students will choose among the various methods for solving quadratic functions, and give reasons for their choices 5. Students will interpret the meaning of intercepts of a quadratic in the context of a real world problem 6. Students will define the absolute value function as a piecewise defined function	○ Activity 2.2.1: Product of Two Lines ○ Activity 2.2.2: Multiplying and Factoring ○ Activity 2.2.3: Solve Equations by Factoring ○ Activity 2.2.4: Absolute Value ○ Activity 2.2.5: Completing the Square ○ Activity 2.2.6: Deriving the Quadratic Formula	Exit Slip 2.2.1 Exit Slip 2.2.2 Exit Slip 2.2.3 Journal Prompt 1 Journal Prompt 2 Journal Prompt 3

	7. Students will determine when a quadratic equation as real number solutions or not		
N.RN.3 N.CN.1 N.CN.2 N.CN.7	8. Identify types of numbers: natural, whole, integers, rational, irrational, real, complex 9. Determine whether a set of numbers is closed under an operation 10. Determine whether or not a number, or sum or product of numbers is rational or irrational 11. State the definition of *i* 12. Express the square root of a negative number in terms of *i* 13. Evaluate i^n for any natural number n 14. Identify the real and imaginary parts of a complex number, using the a+bi notation 15. State the conjugate of a complex number 16. Add, subtract, and multiply complex numbers - Solve quadratic equations that have complex solutions 17. -State the two solutions in a+bi form 18. -Given a single complex solution to a quadratic equation, give the other solution.	○ Activity 2.3.1: Closure and Sets of Numbers ○ Activity 2.3.2: Imaginary Numbers ○ Activity 2.3.3: Complex Numbers ○ Activity 2.3.4: Complex Solutions to Quadratic Equations	Exit Slip 2.3.1 Exit Slip 2.3.2 Exit Slip 2.3.3 Journal Prompt 1 Journal Prompt 2
			Mid Unit Test
N.CN.2 N.CN.9	19. Determine the number of roots of a quadratic equation based on the discriminant 20. Determine the sign of the discriminant of a quadratic equation given the graph	○ Activity 2.4.1: Bungee Jumping ○ Activity 2.4.2: Zeros? Real or Complex? ○ Activity 2.4.3: Complex Zeros ○ Activity 2.4.4: Putting it All Together	Exit Slip 2.4.1 Exit Slip 2.4.2 Journal Prompt 1 Journal Prompt 2

	21. Add, subtract, and multiply complex numbers 22. Demonstrate the Fundamental Theorem of Algebra		
BF.A.1 CED.A.1 CED.A.2 IF.B.4 IF.C.7 IF.C.7A	23. Fit a quadratic function to data that follows a quadratic relationship and use the function to make a prediction 24. Develop a quadratic function using algebraic relationships. 25. Solve contextual problems involving quadratic functions. 26. Solve optimization problems that involve finding the maximum value (or minimum value) of a quadratic function	○ Activity 2.5.1: Home Run Ball ○ Activity 2.5.2: Areas of Equilateral Triangles ○ Activity 2.5.3: Parabolic Behavior ○ Activity 2.5.4: Historic Hotels	Exit Slip 2.5.1 Exit Slip 2.5.2 Exit Slip 2.5.3 Journal Entry 1 Journal Entry 2
REI.A.2 CED.A.1 CED.A.2 IF.B.4 IF.C.7A IF.C.7B	27. Solve equations involving one and multiple square root terms. 28. Solve radical equations graphically. 29. Determine if a solution obtained from an equation-solving process is extraneous. 30. Model and solve contextual problems involving radical equations.	○ Activity 2.6.1: Radical Functions ○ Activity 2.6.2: Solving Radical Equations ○ Activity 2.6.3: Crossing Lake James	Exit Slip 2.6.1 Exit Slip 2.6.2 Exit Slip 2.6.3 Journal Entry 1 Journal Entry 2
			Unit 2 Performance Task
			End of Unit Test

Suggested Resources and Texts: Graph paper, youtube videos, “Three Lessons on Parabolas” by Ben Ceyanes, Pamela Lockwood, Kristina Gill, NCTM Illuminations activity, “Don’t Teach Technology, Teach with Technology.”

Suggested Technology: Graphing calculators, computer software with a graphing utility for all activities, online access, TI emulator, TI-84 TRANSFORM App, Camera

Unit 3: Polynomial Functions

Introduction and Established Goals:

Through their study of quadratic functions, students have begun to recognize properties of the graph of a function such as the extrema, x- and y-intercepts, symmetry of the graph with respect to a line, concavity, and end behavior. The study of polynomial functions extends the study of non-linear functions started with quadratic functions to include functions of the form y = P(x) where P(x) is a polynomial of degree $n \geq 2$.

Mastery of the unit will require students to develop their understanding of: 1) the relationship between zeros of the polynomial function and the factors of the related polynomial; 2) the x- and y-intercepts of the function; 3) the end behavior of the y-values of the function as x approaches positive or negative infinity; 4) the transformations of the graph of *y=f(x)* when it is reflected over the x- and y-axes and under what conditions will the function be odd or even, i.e. when *f(-x) =- f(x)* or when *f(-x) = f(x)*; 5) relative extrema of polynomial functions and how to interpret such values in the context of real-world problems; 6) operations involving polynomial expressions, including division of polynomials and the connection of division to the Division Algorithm, the Remainder Theorem, and the Factor Theorem; 7) factoring polynomials by various methods including the application of the Remainder Theorem and advanced polynomial identities; 8) the connection between Binomial Expansion and Pascal's Triangle; 9) applications of polynomials in modeling real-world functions, including solving counting problems by using generating functions; and 10) the difference between exponential growth versus growth modeled by polynomial functions.

The unit includes six investigations through which the topics will be developed. Each investigation has several activities that develop the objectives of the unit. The activities are designed to allow students to explore relationships and formulate conjectures related to the basic principles in working with polynomials. Technology is used in many activities to allow students to manipulate parameters in a function in order to discover patterns in the behavior of the graph of the function. Most activities are written to enable students to do use any graphing utility available in their classroom, but many activities were written using GeoGebra since the algebraic, graphic, and numerical representations of the polynomial functions are readily available.

Desired Outcome(s):

- The exponential growth family will always overtake the polynomial family no matter how large the degree or how large the leading coefficient.
- The end behavior of a polynomial is determined by its degree, even or odd, and the sign of its leading coefficient.
- The solutions or roots of the polynomial equation $y = a_n x^n + a_{n-1} x^{n-1} + \ldots + ax_1 + a_0$ are the x coordinates of the x-intercepts of $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + ax_1 + a_0$ and are the zeros of the function f(x).
- Long division by the binomial *x–a* can be used to factor a polynomial and the Remainder = P(a).

CT State Standard(s):

F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; multiplicity of roots; symmetries; end behavior; and periodicity.

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.APR.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2– y^2)\bullet (x^2 + y^2)$.

A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

A.APR.5 (+) Know and apply that the Binomial Theorem gives the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.) For example given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

SL.9-10.1.c Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions.

Common Core Standard(s):

- MP 2: Reason abstractly and quantitatively
- MP 4: Model with mathematics
- MP 7: Look for and make use of structure.

Essential Question(s):

- What are the basic features of a polynomial function based on the degree of the polynomial?
- How is the division of a polynomial, *P(x)*, by a binomial of the form $x – a$ connected to the polynomial function, $y = P(x)$, when evaluated at $x=a$?
- How can long division by the binomial $x–a$ be used to factor a polynomial?
- What is the connection between the zeros of a polynomial function, the x-intercepts of the graph of the polynomial function, and the factors of the polynomial?
- How is the expansion of the binomial $(a+b)^n$ connected to Pascal's Triangle?

- How can polynomial functions be applied to applications in combinatorics and mathematical models?
- How does polynomial growth compare to exponential growth?

Key Terms/Concepts:
Algebraic identity, Array, Binomial Theorem, Coefficient, Combinations, Complex coefficient, Compound interest, Decreasing function, Degree, Division algorithm, Divisor, End behavior, Even function, Exponential Function/Growth, Extrema, Factor theorem, Generating Polynomial/function, Increasing function, Linear factor, Mathematical model, Maximum, Minimum, Odd function, Pascal's triangle, Polynomial, Power, Quadratic factor, Quotient, Reflection, Regression equations, Remainder, Remainder theorem, Root, Scatter plots, Series, Subsets, Transformation x-intercepts, y-intercepts, Zero, Zero of multiplicity, Zero product property.

LEARNING PLAN

STANDARD	LEARNING OBJECTIVES (Content and Skill)	INSTRUCTIONAL STRATEGIES	ASSESMENT EVIDENCE
F.IF.7C F.IF.7 F.IF.4	1. Determine the basic shape and end behavior of the graph of a polynomial function based on the term of highest degree of the polynomial. 2. Determine the x- and y-intercepts of a polynomial function by inspection of the equation of the function when the polynomial is in factored form. 3. Identify extrema of a polynomial function given the graphic, symbolic, or numerical representations of the polynomial. 4. Interpret the meaning of intercepts and extrema of a polynomial in the context of a real-world problem. 5. Determine if a graph of a polynomial function is tangent to the x-axis or crosses the x-axis depending on the multiplicity of the corresponding linear factor of the function.	○ Activity 3.1.1: Winter in CT ○ Activity 3.1.2: Polynomial Investigation – end behavior ○ Activity 3.1.3: Polynomial Investigation – Odd and Even Functions ○ Activity 3.1.4: The Open Box ○ Activity 3.1.5: Polynomial Functions with Repeated Factors ○ Activity 3.1.6: Polynomial Functions in Action	Exit Slip3.1.1 Exit Slip 3.1.2 Exit Slip 3.1.3 Exit Slip 3.1.4 Journal Prompt 1 Journal Prompt 2
A.APR.1	6. Add, subtract, and multiply polynomials.	○ Activity 3.2.1: Roller Coasters and Curves	Exit 3.2

A.APR.2	7. Divide polynomials using long division. 8. State the Remainder Theorem and its implications. 9. Determine if (x–a) is a factor of P(x) by either calculating the remainder using long division or finding the value of P(a).	○ Activity 3.2.2: Roots and Factored Form of Polynomials ○ Activity 3.2.3: Polynomial Long Division and the Remainder Theorem ○ Activity 3.2.4: Finding an Equation vs. Finding the Equation ○ Activity 3.2.5: Identities Galore: Summing it Up! ○ Activity 3.2.6: That Sums it Up!	Journal Prompt 1 Journal Prompt 2 Journal Prompt 3
A.APR.5	10. Construct the array of numbers known as Pascal's Triangle and know that the entries in the nth line in the array represent the number of subsets containing 0, 1, 2, ... , n–1 elements from left to right. 11. Apply the Binomial Theorem to expand any power of a binomial expression of the form (x + y)n. 12. Apply the binomial expansion to compute the number of combinations of n objects taken r at a time.	○ Activity 3.4.1: False Advertising ○ Activity 3.4.2: Pascal's Pizza Parlor ○ Activity 3.4.3: Pascal's Triangle	Exit Slip 3.4 Journal Prompt 1 Journal Prompt 2
A.CED.2	13. Create a polynomial function model from real-life data using higher order polynomials using regression features of a graphing utility. 14. Interpret the meaning of characteristics of a polynomial function in the context of a real-world problem. 15. Interpret the meaning of the coefficients of a polynomial that is the result of the product in a generating function.	○ Activity 3.5.1: Technology Companies ○ Activity 3.5.2: Roller Coaster Redux ○ Activity 3.5.3: Can You Count It?	Exit Slip 3.5.1 Journal Prompt 1 Journal Prompt 2 Journal Prompt 3
F.IF.9 F.LE.3	16. Construct and compare exponential and polynomial function models.	○ Activity 3.6.1: How Fast Will it Grow? ○ Activity 3.6.2: Exponential vs Polynomial	Exit Slip 3.6.1 Exit Slip 3.6.2

	17. Solve problems based on exponential and polynomial function models. 18. Interpret whether a polynomial function or an exponential function best models a real-world relationship. 19. Recognize that quantities growing exponentially will exceed the growth of functions modeled with polynomial functions as x increases to ∞.	○ Activity 3.6.3: Why Does Exponential Growth Always Surpass Polynomial?	Journal Prompt 1 Journal Prompt 2
			Unit 3 Performance Task
			End of Unit Test

Suggested Resources and Texts: http://www.epsilon-delta.org/2012/02/end-behavior-activities.html.

http://www.gilbertschools.net/cms/lib3/AZ01001722/Centricity/Domain/819/Algebra II Teachers Textbook Chapter 5.pdf

http://www.gilbertschools.net/cms/lib3/AZ01001722/Centricity/Domain/819/Algebra II Teachers Textbook Chapter 4.pdf

http://www.gilbertschools.net/cms/lib3/AZ01001722/Centricity/Domain/819/Algebra II Teachers Textbook Chapter 3.pdf

Data Source: Electronic Arts, Inc., Annual Reports, March 31, 2002 and 2004, retrieved from http://dufu.math.ncu.edu.tw/calculus/calculus_pre/node7.html, January 6, 2015

Evered, Lisa J. and Brian Schroeder. 1991. "Counting with Generating Functions." In *Discrete mathematics across the Curriculum: K-12.* 1991 Yearbook of the National Council of Teachers of Mathematics, edited by Margaret J. Kenney and Christian R. Hirsch. Reston, VA: NCTM, 1991, 143-148.

Suggested Technology: Geogebra Software

Unit 4: Rational and Power Functions

Introduction and Established Goals:
This unit provides a study of rational functions, building on the students' familiarity with other function families (linear in algebra 1, quadratic in algebra 1 and unit 2 of algebra 2, polynomial in algebra 2 and exponential in algebra 1). Rational functions are introduced in the context of a science experiment that discovers the Inverse Square Law (for Light) and hence requires the study a new function family and a review of integral exponents. Students will later use data that they have researched on the internet and then model, using the power regression on a graph or other available technology. This will provide the motivation to review and extend their earlier work in unit 7 of algebra 1 with rational exponents and the work done in unit one of this course with radical notation. A second modeling example will require a negative rational exponent. Modeling will require students to problem solve, research, and use a discovery approach to learning key features and behavior of simple rational functions. Students will explore the graphs of the power family and the roles of the parameters *a* and *p* in $f(x) = ax^p$. Selected members of the rational family will be explored so that end behavior can be determined, vertical asymptotes and location of holes can be determined and students can determine if a graph will cross its horizontal asymptote. Students will also add, subtract, multiply and divide rational expressions so they can see that rational expressions form a system analogous to the rational numbers. They will solve rational equations.

By the conclusion of the unit they will see that polynomial functions are a subset of the rational function family and that some rational functions are also members of the power family. The power family permits students to look back upon linear functions in the guise of direct variation and the power family and simple rational family in the guise of indirect variation and then to extend both to other variation situations, for example, the area of a circle is directly related to the square of the radius. The interrelationship between multiple representations verbal (words), numerical or tabular, graphical, and symbolic (equations) will continue to be emphasized. Comparisons and contrasts with all functions studied to date will continue to be made and by the end of the unit students should see that the polynomial function family is built on linear and quadratic functions, and that the rational family is built on the polynomial family. Students will continue to be asked to defend their statements, to formulate definitions and to use precise mathematical language and appropriate symbols.

Desired Outcome(s):

- Some functions have vertical asymptotes and unique behavior near one
- Why do some functions have horizontal asymptotes?
- What is the significance of a vertical or horizontal asymptote especially in a contextual setting?
- Why can rational equations have extraneous roots?
- How does inverse variation differ from direct variation?
- How does a power function differ from polynomial functions with only one term?

CT State Standard(s):

A.SSE.1	Interpret expressions that represent a quantity in terms of its context.
A.SSE.1b	Interpret complicated expressions by viewing one or more parts as a single entity.
F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship.
F.IF.5	Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.7d(+)	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, by table, or verbally)
F.BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
A.CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable in a modeling context.
A.APR.1b	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for more complicated examples, a computer algebra system.
A.APR.6	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for more complicated examples, a computer algebra system.
A.APR.7(+)	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous roots may arise.
A.REI.11b	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equations f(x) = g(x); find the solutions approximately, e.g. using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
SL.9-10.1.c	Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions.

Common Core Standard(s):

- MP1: Make sense of problems and persevere in solving them
- MP3: Construct viable arguments and critique the reasoning of others
- MP4: Model with mathematics

Essential Question(s):

- What is the advantage of using a function or an equation to model a real world relationship?
- What is the advantage of using a graphical representation of a function to model a real world relationship?
- Why do some mathematical models have limitations when used to model a real world situation?
- How do the mathematical models you have studied so far differ from one another? How are they similar?
- When you are deciding upon a mathematical model to use, what factors must you consider?

Key Terms/Concepts:

Luminosity, Asymptote, Area, Brightness, Combined Variation, (Dis)continuous, Domain, Degree, Direct Variation, Distance, Excluded Values, Extraneous solution, Equivalent expressions, end behavior, factors, indirect variation, inverse square law, joint variation, outlier, proportional, parameter, power regression, power function family, rational function, range, ratio, scaling factor

LEARNING PLAN

STANDARD	LEARNING OBJECTIVES (Content and Skill)	INSTRUCTIONAL STRATEGIES	ASSESMENT EVIDENCE
A.SSE.1 N.RN.1 N.RN.2 F.IF.4 F.IF.5 F.IF.7 F.IF.9 A.CED.3 A.REI.10 A.CED.1	1. Distinguish, given a graph, linear vs nonlinear behavior. 2. Distinguish, given a graph, differing nonlinear behaviors, in particular between polynomial degree ≥ 3 and quadratic, exponential, and now the new families--the power and selected rational. 3. Explain how indirect variation differs from direct variation by equation and graph. 4. Model situations with a direct or indirect variation function as appropriate. 5. Demonstrate that the brightness of a source of light is a function of the inverse square of its distance. 6. Make a quick sketch of f(x) = kx-n for n = 1, 2, 3, 4. 7. Describe in words the end behavior of f(x) = kx-n, n a natural number, and the behavior near x = 0. Later, for n a rational number.	○ Activity Lab Sheet 4.1.1 ○ Activity 4.1.2: Evaluating and Graphing ○ Activity 4.1.3: Direct/Indirect Variation ○ Activity 4.1.4: Other Applications of an Inverse Square Law ○ Activity 4.1.5: Evaluating and Graphing ○ Activity 4.1.6: A Review of Exponents ○ Activity 4.1.7: Collecting Bird Data	Exit Slip 4.1.1 Exit Slip 4.1.2 Journal Prompt 1 Journal Prompt 2
A.SSE.1 N.RN.1 N.RN.2 F.IF.4 F.IF.5	8. Describe the differing behaviors of the graphs defined by f(x) = axp. 9. Explain the role of the parameter a in f(x) = axp and the role of p when p > 0 and when p < 0 for 0 < x < 1 and x ≥ 1.	○ Activity 4.2.1: Analysis of Bird Data ○ Activity 4.2.2: Application Problems ○ Activity 4.2.3: Pace vs. Speed ○ Activity 4.2.4: Graphing Investigation ○ Activity 4.2.5: Variation Application Problems	Exit Slip 4.2.1 Exit Slip 4.2.2 Exit Slip 4.2.3 Journal Prompt 1 Journal Prompt 2

F.IF.7 A.CED.1 A.CED.3 A.REI.10	10. Explain why f(x) = axp also models a simple rational function when p is a negative integer and determine the domain, and equations of any vertical and horizontal asymptotes. 11. Explain why f(x) = axp also models a simple polynomial function when p is a positive integer. 12. Compare power, linear, and exponential functions. 13. Model inverse variation, direct variation, joint or combined variation situations as appropriate by an equation.		
A.SSE.1B A.APR.1B A.APR.7 F.IF.4 F.IF.5 F.IF.7 F.IF.7D F.IF.9 F.BF.3 A.REI.10	14. Explain why the graphs of some rational functions have horizontal asymptotes. 15. Determine the equation of a horizontal asymptote when it exists. 16. Explain why the graphs of some rational functions have vertical asymptotes. 17. Explain why the graphs of some rational functions have a hole. (+) 18. Determine the equation of a vertical asymptote. 19. Interpret the meaning of a vertical or horizontal asymptote in a real world setting. 20. Determine the location of a hole in a graph. (+) 21. Make quick sketches of rational functions of the form f(x) = (ax + b) /(cx + d).	○ Activity 4.3.1: Graphing Rational Functions I ○ Activity 4.3.2: Graphing Rational Functions II ○ Activity 4.3.3: Graphing Rational Functions III ○ Activity 4.3.4: Applications of Rational Functions ○ Activity 4.3.5: Graphing Rational Functions IV	Exit slip 4.3.1 Exit slip 4.3.2 Journal Prompt 1 Journal Prompt 2
A.SSE.1B	22. Translate verbal descriptions of	○ Activity 4.4.1: Rational Expressions I	Exit 4.4.1

A.APR.6 A.APR.7	relationships that need rational expressions. 23. Use technology to solve problems that utilize rational expressions or rational functions.	○ Activity 4.4.2: Rational Expressions II ○ Activity 4.4.3: Rational Expressions III	Exit 4.4.2 Journal Prompt 1 Journal Prompt 2 Journal Prompt 3
A.SSE.1B A.REI.2 A.REI.11 A.CED.1	24. Interpret and solve an application problem that can be modeled by a rational or power function. 25. Verify that a solution makes sense in the context of the problem. 26. Check proposed solutions to be sure no root is an extraneous one. 27. Translate a verbal description of a relationship into an equation.	○ Activity 4.5.1: More Equations and Problems ○ Activity 4.5.2: Solving Equations with Fractions and Rational Expressions ○ Activity 4.5.3: More Equations and Problems	Exit Slip 4.5 Journal Prompt 1 Journal Prompt 2
			Unit 4 Performance Task
			End of Unit Test

Suggested Resources and Texts: Ruler with centimeter markings, Graph paper for activity 4.1.1, Flash lights mini Maglite recommended for activity 4.1.1, Cardboard for activity 4.1.1, Heavy construction paper or poster board for activity 4.1.1, Scissors or an Exacto knife for activity 4.1.1, Transparent tape for activity 4.1.1, Graph paper for all activities, Mathematics Teacher "Queueing Theory: A Rational Approach to the Problem of Waiting in Line V5, No. 5 , May 2012, https://www.youtube.com/watch?v=fG-zjUR9mM8, www.youtube.com/watch?V=5BVSRj_ZEuM.

Suggested Technology: Graphing calculator/computer software with a graphing utility for all activities

Unit 5: Exponential and Logarithmic Functions

Introduction and Established Goals:
The Algebra 2 unit on exponential and logarithmic functions builds upon the concepts explored in Algebra 1 Unit 7 and Algebra 2 Unit 1. Instructors are encouraged to revisit the core concepts of those units, especially Algebra I Unit 7, as substantial time may have elapsed since that material was covered or it may have been omitted completely. The investigation overviews will recommend specific investigations from the Algebra 1 curriculum and activities within those investigations.

Building upon the concept of a geometric sequence, characterized by a common quotient, students develop the continuous exponential function and in particular the natural base *e*. Examination of the graph of $f(x) = ab^x$ will lead students to conclusions regarding the possible values of b and a, as well as the domain and range of the function, and that the function is in fact an invertible function. All but the latter were investigated in Algebra 1 Unit 7. The definition of the appropriate inverse function g(x) = $\log_b x$ and the assumption of continuity will permit students to develop methods for solving exponential and logarithmic equations.

Initially, the focus of this unit should be on the graphs of the logarithmic and exponential functions, and answering fundamental questions regarding them. If for example, $f(x) = 2^x$ is assumed to be continuous, students will readily recognize that the solution to $8 = 2^x$ is the x-value of the common point of intersection of the graphs of $y = 2^x$ and $y = 8$. Or they can observe that they are looking for the input that produces an output of 8. A richer development results from considering the intersection of $y = 2^x$ and a horizontal line whose height is not an integral power of 2, such as, $y = 14$. Under the assumption of continuity for $y = 2^x$ it is graphically clear that an intersection exists and that therefore $14 = 2^x$ has a solution. Moreover, encouraging the student to describe this solution *in words*, without concern for a specific value, leads to the statement that *"x is the exponent to which 2 is raised to produce 14"* and the translation to $x = \log_2 14$. Building along these lines leads to the definition of the inverse function, $g(x) = \log_b x$. Remind them that as in Unit 1 Investigation 6, when f and f^{-1} exist, f(c) = d is equivalent to $f^{-1}(d) = c$ and stating this definition in terms of the exponential and the new logarithm function $f(x) = b^x$ is equivalent to $f^{-1}(x) = \log_b x$. Emphasize that a logarithm is an exponent.

The development of the logarithmic function as the inverse of the exponential function and the understanding that a logarithm is an exponent will assist students in discovering the rules for logarithms. For example, students can be asked to find $\log_2 4 + \log_2 8$, and compare this result to $\log_2 32$. With a sufficient number of examples and some prompting to look for a relationship between the numbers *m*, *n* and *z(=mn)* the "Product Rule" $\log_b z = \log_b m + \log_b n$ can be established and the "Power Rule" is seen to be a special case by noting: $\log_b m^t = \log_b mmmm \ldots m = \log_b m + \log_b m + \cdots \log_b m = t \log_b m$, *t* a Real number. Of course *m* and *n* must both be positive. The quotient rule follows easily from these results.

Applications and the multiple real world situations where logarithms are needed are investigated: sound and the decibel, earthquakes and their scales, measuring PH, brightness of stars, continuous compounding and other financial applications provide a rich and interesting source that avoids the student asking, "Where am I ever going to use this?"

The interrelationship between multiple representations verbal (words), numerical or tabular, graphical, and symbolic (equations) continues to be emphasized. Students will continue to be asked to defend their statements, to formulate definitions and to use precise mathematical language and appropriate symbols.

The Exponential/Logarithm unit consists of 7 investigations of which 1, 2, 4, 5 and 6 form the essential core needed for further study. Instructors pressed for time should ensure that these investigations are covered thoroughly. Investigations 3 and 7 are valuable extensions for all citizens in today's world but some of the activities can be omitted in the interest of time. The investigation overviews will provide further information.

Desired Outcome(s):

- When a function is one-to one we can undo it and define an inverse function.
- The logarithmic form can always be rewritten in exponential form and vice versa.
- A logarithm is an exponent.
- The laws of exponents make the laws of logarithms sensible and easy to remember.
- With real data, sometimes deciding whether data is linear or non-linear is more complex than just looking at a graph, differences ($y_n - y_{n-1}$), or an r-value; it is important to examine differences that are approximately the same more carefully to see if there is a pattern of increasing or decreasing values that, because the pattern is exponential, soon begin to produce outputs with extremely large or small values.
- An informal limiting process permits us obtain a continuous compounding formula from the finite compounding formula and define the number *e*.
- The domain of a logarithmic family member guarantees that the argument of the function is nonnegative.
- A logarithmic scale is needed when the numbers we need to graph vary greatly in size.
- The logarithmic and exponential families are rich in applications to the real world.

CT State Standard(s):

A.SSE.1	Interpret expressions that represent a quantity in terms of its context.
A-SSE-1b	Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as a product of P and a factor not depending on P.
A-SSE-4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.
A-CED-1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and <u>exponential</u> functions.*
A-CED-2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
F-IF. 7e	<u>Graph exponential and logarithmic functions, showing intercepts and end behavior</u>, and trigonometric functions, showing period, midline, and amplitude.
F-IF-8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F.BF.1	Write a function that describes a relationship between two quantities.
F-BF-3	Identify the effect on the graph of replacing $f(x)$ by $fk, kf(x), f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the values of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
F.BF.4a	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2(x^3)$ or f(x) = (x + 1)/(x – 1) for x ≠ 1.
F.BF.4b (+)	Verify by composition that one function is the inverse of another.
F.BF.4c (+)	Read values of an inverse function from a graph or a table, given that the graph has an inverse.

F-BF-5 (+)	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents**.**
F-LE-4	For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where a, c and d are numbers and the base b is 2, 10 or *e;* evaluate the logarithm using technology.
F-LE-5	Interpret the parameters of an exponential function in terms of a context.
A.REI.11b	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equations f(x) = g(x); find the solutions approximately, e.g. using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
SL.9-10.1.c	Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions.

Common Core Standard(s):

- MP3: Construct viable arguments and critique the reasoning of others
- MP4: Model with mathematics
- MP6: Attend to precision

Essential Question(s):

- Why do some mathematical models have limitations when used to model a real world situation?
- When you are deciding upon a mathematical model to use, what factors must you consider?
- How do you know that the exponential function is invertible?
- What characterizes logarithmic growth?
- What characterizes exponential growth and decay?
- What are real world models of exponential and logarithmic growth and decay?
- What are the limitations of exponential growth models?
- How can one differentiate an exponential model from a linear model given a real world data set?

Key Terms/Concepts:
Base e, Change of Base Rule, Common quotient, compound interest, continuous compounding, exponential function base e, finite sum, geometric series, growth factor, inverse function, invertible, logarithmic function, natural logarithm, one to one function, power rule, product rule, quotient rule, strictly decreasing, strictly increasing.

LEARNING PLAN

STANDARD	LEARNING OBJECTIVES (Content and Skill)	INSTRUCTIONAL STRATEGIES	ASSESEMENT EVIDENCE
F.IF.7E	1. Graph exponential functions and determine their properties.	○ Activity 5.1.1: New Beginnings	Exit Slip 5.1.1
F.BF.5		○ Activity 5.1.2: Using the Definition	Exit Slip 5.1.2
F.BF.4A	2. Graph logarithmic functions as inverses of exponential functions, and determine their properties.	○ Activity 5.1.3: Exploring Log Functions	Journal Prompt 1
F.BF.4B		○ Activity 5.1.4: The Product and Quotient Rules	
		○ Activity 5.1.5: The Power Rule	

F.BF.4C	3. Describe the domain of $f(x) = lnx$, and determine which domain values give $f(x) < 0$; $f(x) = 0$; $and\ f(x) > 0$ **4.** Demonstrate graphically, numerically and algebraically the inverse relationship between the exponential family and the logarithmic family. **5.** Solve logarithmic problems by rewriting as exponential problems. 6. Interpret and solve an application problem that can be modeled by an exponential or logarithmic function. 7. Verify that a solution makes sense in the context of the problem.	○ Activity 5.1.6: How High is that Stack of Paper? ○ Activity 5.1.7: Logarithmic Scavenger Hunt ○ Activity 5.1.8: Consequences of Being Inverse Functions	
F.BF.5 A.REI.2	8. Model compound interest with an appropriate formula. 9. Distinguish between situations that need finite compound interest and those that need continuous compounding. 10. Approximate ln *c* with an integer and using technology obtain a decimal approximation. 11. Solve $e^x = c, where\ c > 0$ by writing as $ln(c) = x$ and then approximating *x* with technology; recognize that for $c \leq 0$ the equation has no solution. 12. Solve $ae^{kt} = c$ for *t* by rewriting as $t = \frac{ln\left(\frac{c}{a}\right)}{k}$ and approximating appropriately with technology.	○ Activity 5.2.1: How Many Compounding Periods? ○ Activity 5.2.2: Revisiting e and compound interest ○ Activity 5.2.3: The Remarkable e ○ Activity 5.2.4: Equations Involving Logarithms	Exit Slip 5.2.1 Exit Slip 5.2.2 Journal Prompt 1

	13. Solve some applied problems such as finding a doubling time, half-life and other problems including financial ones.		
F.LE.4	14. Use properties of logarithms to solve equations. 15. Use a logarithmic scale. 16. Use formulas that have logarithms in them. 17. Apply their knowledge of logarithmic scales to real world applications.	○ Activity 5.3.1: Can we eat the Chicken? ○ Activity 5.3.2: Earthquakes ○ Activity 5.3.3: Basic or Acidic ○ Activity 5.3.4: Measuring Sound Intensity ○ Activity 5.3.5: Problems using Logarithmic Scales.	Exit Slip 5.3 Journal Prompt 1
A.SSE.1B F.IF.8 F.LE.4 F.BF.3 F.LE.5	**18.** Students will be able to apply their knowledge of the roles of k, d and c to graph y = kg(x), y = g(kx), y = d + g(x) and y = g(x + c) given $f(x) = ab^x$. and to obtain an equation given a graph. **19.** Students will apply the rules from investigation 5.1 to base e and note that in particular ln e^x = x and $e^{\ln x}$ = x. **20.** Students will understand why and apply *Any exponential function can be written in either of two forms y = ab*^{x} *and y = ae*^{rx} *where r = ln b.* 21. Students will be able to solve selected applications, including continuous compound interest problems using the laws of logarithms and their ability to solve $ab^{(ct)} = d$ using logarithms.	○ Activity 5.4.1: Changing Parameters ○ Activity 5.4.2: Transformations ○ Activity 5.4.3: Applications	Exit Slip 5.4.1 Exit Slip 5.4.2 Journal Prompt 1
A.CED.1 A.CED.2 F.BF.3	**22.** Students will be able to apply their knowledge of the roles of k, d and c to graph y = kg(x), y = g(kx), y = d + g(x) and y = g(x +	○ Activity 5.5.1: US Census ○ Activity 5.5.2: Modeling the US Population ○ Activity 5.5.3: Superbugs and the Spread of Diseases	Exit Slip 5.5 Journal Prompt 1

F.LE.5	c) given $f(x) = ab^x$. and to obtain an equation given a graph. **23.** Students will apply the rules from investigation 5.1 to base e and note that in particular $\ln e^x = x$ and $e^{\ln x} = x$. **24.** Students will understand why and apply *Any exponential function can be written in either of two forms* $y = ab^x$ *and* $y = ae^{rx}$ *where* $r = \ln b$. 25. Students will be able to solve selected applications, including continuous compound interest problems using the laws of logarithms and their ability to solve $ab^{(ct)} = d$ using logarithms.	○ Activity 5.5.4: The Cost of a Used Car ○ Activity 5.5.5: Which Model Should I Use? ○ Activity 5.5.6: Fitting an Exponential Function by Linearizing the Data	
A.CED.1 A.CED.2 F.LE.5 A.SSE.4	26. Interpret and solve application problems that can be modeled by various financial formulas. 27. Verify that a solution makes sense in the context of the problem. 28. Be able to explain the power of exponential growth that comes with time. 29. Translate a verbal description of a relationship into an equation or an inequality.	○ Activity 5.7.1: Saving for a Down Payment I ○ Activity 5.7.2: Think Before You Buy that Drink ○ Activity 5.7.3: Saving for a Down Payment II ○ Activity 5.7.4: Mortgage Payments ○ Activity 5.7.5: What Car Can I Afford? ○ Activity 5.7.6: Financing Options ○ Activity 5.7.7: Investment Opportunities	Exit Slip 5.7.1 Exit Slip 5.7.2 Journal Prompt 1 Journal Prompt 2
			Unit 5 Performance Task
			End of Unit Test

Suggested Resources and Texts:

http://en.wikipedia.org/wiki/Britney_Gallivan

http://www-history.mcs.st-andrews.ac.uk/Biographies/Napier.html https://www.youtube.com/watch?v=KRAEbotuIE which taped the paper-folding of a piece the size of a football field for MYTHbusters Folding Paper Seven Plus Times.

www.youtube.com/watch?v=b-MZumdfbt8

http://www-history.mcs.st-and.ac.uk/PrintHT/e.html

June 2015 Smithsonian Magazine ,"Aftershock", pages 36 – 43 by Elizabeth Kolbert

Suggested Technology: Calculator, internet access, graphing software

Unit 6: Sequences and Series

Introduction and Established Goals: This chapter builds on skills from Algebra 1, where arithmetic and geometric sequences were first introduced. The first three lessons review and extend work students have previously done. New in this chapter is the skill of adding terms of a sequence. Partial sums and sums of infinite geometric series are explored numerically and graphically. The final lesion in the chapter involves recursively defined functions. This reviews knowledge of arithmetic and geometric sequences. Connections to linear and exponential functions are made.

Desired Outcome(s):

- Differentiate between an arithmetic and geometric sequence
- Use explicit and recursive rules to find the *n*th term of a sequence
- Find partial sums of sequences
- Find sums of infinite geometric sequences

CT State Standard(s):

F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

A-SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

F-BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

SL.9-10.1.c Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions.

Common Core Standard(s):

- MP2: Use numbers to help solve problems
- MP8: Use mathematical patterns to help me solve a problem

Essential Question(s):

- How can you write a rule for the *n*th term of a sequence?
- How can you recognize an arithmetic sequence from its graph?
- How can you recognize a geometric sequence from its graph?
- How can you find the sum of an infinite geometric series?
- How can you define a sequence recursively?

Key Terms/Concepts: Sequence, Terms of a sequence, Series, Summation notation, Sigma notation, Domain, Range, Arithmetic sequence, Common difference, Arithmetic series, Linear function, Mean, Partial sum, Repeating decimal, Fraction in simplest form, Rational number, explicit rule, recursive rule

LEARNING PLAN

STANDARD	LEARNING OBJECTIVES (Content and Skill)	INSTRUCTIONAL STRATEGIES	ASSESMENT EVIDENCE
F-IF.A.3 SL.9-10.1.c	1. Use sequence notation to write terms of sequences 2. Write a rule for the *n*th term of a sequence 3. Sum the terms of a sequence to obtain a series and use summation notation	○ Examples 8.1 ○ Exercises 8.1	Exit Slip 8.1 Homework
F-IF.A.3 F-BF.A.2 F-LE.A.2 SL.9-10.1.c	4. Identify arithmetic sequences 5. Write rules for arithmetic sequences 6. Find sums of finite arithmetic sequences	○ Explorations 1 and 2 ○ Examples 8.2 ○ Exercises 8.2	Exit Slip 8.2 Homework
A-SSE.B.4 F-IF.A.3 F-BF.A.2 F-LE.A.2 SL.9-10.1.c	7. Identify geometric sequences 8. Write rules for geometric sequences 9. Find sums of finite geometric sequences	○ Explorations 1 and 2 ○ Examples 8.3 ○ Exercises 8.3	Exit Slip 8.3 Homework
			Section 8.1-.3 Quiz
A-SSE.B.4 SL.9-10.1.c	10. Find partial sums of infinite geometric series 11. Find sums of infinite geometric series	○ Explorations 1, 2, and 3 ○ Examples 8.4 ○ Exercises 8.4	Exit Slip 8.4 Homework
F-IF.A.3 F-BF.A.1a F-BF.A.2 SL.9-10.1.c	12. Evaluate recursive rules for sequences 13. Write recursive rules for sequences 14. Translate between recursive and explicit rules for sequences 15. Use recursive rules to solve real-life Problems	○ Explorations 1, 2, and 3 ○ Examples 8.5 ○ Exercises 8.5	Exit Slip 8.5 Homework
			Chapter Review
			Performance Task
			End of Unit Test

Suggested Resources and Texts: BigIdeasMath.com

Suggested Technology: Graphing calculator, Access to internet, Microsoft Excel, Projector